Constrained Variational Formulations via Modified Lagrange Multipliers

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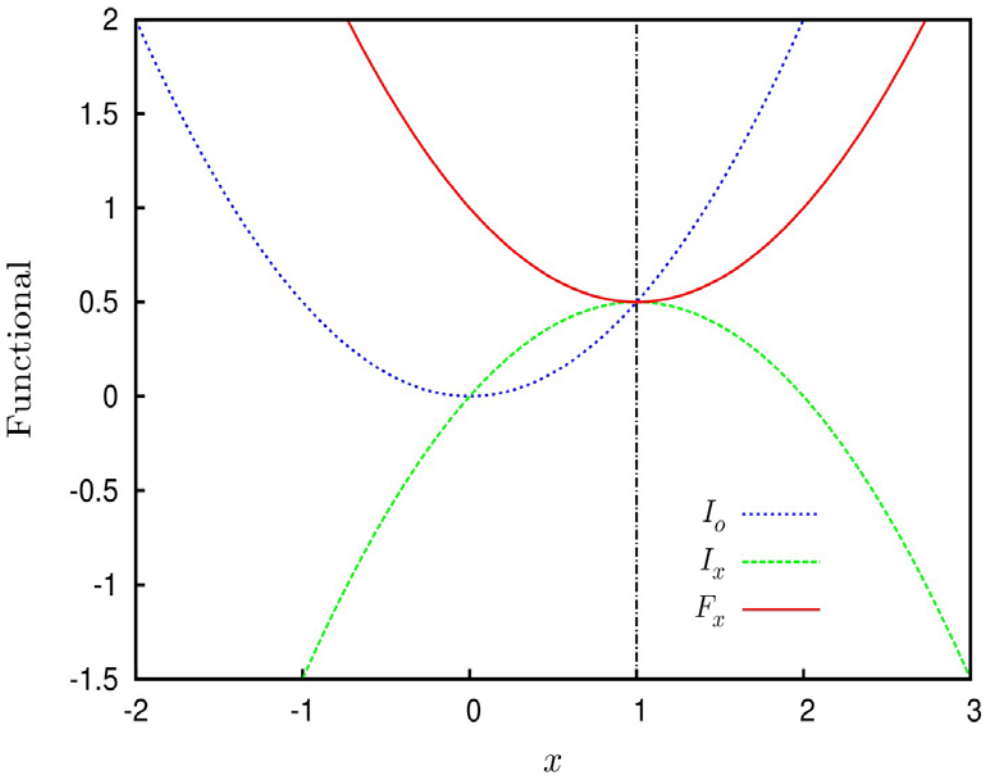
Variational principles are important in the investigation of large classes of physical systems. They can be used both as analytical methods as well as starting points for the formulation of powerful computational techniques. In many cases, the variational principles describing the system are required to obey a number of constraints. These constraints are implemented withing the variational formulation by means of Lagrange multipliers. Often, it is required that the resulting variational functional be positive-definite, in other words, its extremum be actual minimum. In this article, we present a general approach to attack the problem of finding, among equivalent variational functionals, those that generate true minima. The method is based on the modification of the Lagrange multiplier . We apply the method to different examples and, in particular, show how to obtain variational formulations for Poisson and Poisson-Boltzmann systems with extrema that are always minima.



The original (unconstrained) functional is represented by a dotted blue line. The reduced functional produced via the standard Lagrange multiplier method is the dashed green line. The solid red line is the functional that results from the application of the modified Lagrange multiplier procedure. The black dashed-dotted line is the constraint.

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