Coulomb energy of uniformly charged spheroidal shell systems

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We provide exact expressions for the electrostatic energy of uniformly charged prolate and oblate spheroidal shells. We find that uniformly charged prolate spheroids of eccentricity greater than 0.9 have lower Coulomb energy than a sphere of the same area. For the volume-constrained case, we find that a sphere has the highest Coulomb energy among all spheroidal shells. Further, we derive the change in the Coulomb energy of a uniformly charged shell due to small, area-conserving perturbations on the spherical shape. Our perturbation calculations show that buckling-type deformations on a sphere can lower the Coulomb energy. Finally, in order to address the effects of counterion condensation on our results, we employ a Manning-Oosawa two-state model approximation to evaluate the renormalized charge and equilibrium free energy as a function of the shell's aspect ratio. Counterion condensation is seen to favor the formation of spheroidal structures over a sphere of equal area for high values of shell volume fractions. We anticipate our results to add to the theoretical foundation required to understand the control of spheroidal shapes in materials using Coulomb interactions.

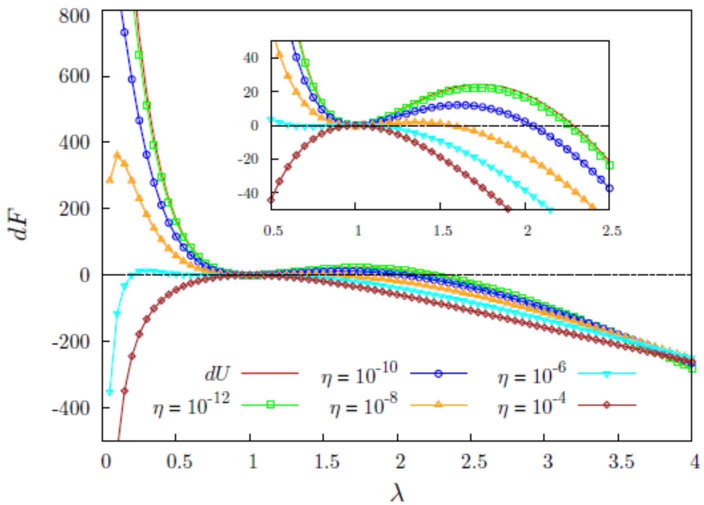

Figure: equilibrium free energy difference, dF, between a homogeneously charged spheroidal shell and a sphere of identical parameters as a function of the aspect ratio λ. The red curve is the case for the counterion-free system. The shell volume fraction η ranges from 10^{-12} (green) to 10^{-4} (brown). The spherical shape becomes a free-energy maximum at $\eta = 10^{-4}$.

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